Optimal trajectories for a quadruped robot with Trot, Amble, Curvet Gaits for two energetic criteria

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Abstract:

In this paper, optimal cyclic reference trajectories are designed for three gaits of a quadruped robot, the *curvet*, the *amble*, and the *trot*, taking into account the actuators characteristics. The gaits are composed of stance phases and instantaneous double supports. The principle of virtual leg is used to obtain simpler dynamic model describing the motion of the quadruped. The impact phases are modeled by passive impact equations. For the *curvet* the step is composed of two different half steps. For the *amble* and *trot* gaits two following half steps are symmetrical.

The optimization problem is solved with an algebraic optimization technique. The actuated joint evolution is chosen as a polynomial function of time. The coefficients of the polynomial functions are optimization parameters. The quadruped studied has non-actuated ankles. The kinetic momentum theorem permits to define the evolution of this non-actuated variable in function of the actuated variables. Two energetic criteria are defined: a *torque cost* and an *energetic cost*. The first is represented by the integral of the torque norm and the second by the absolute value integral of the external forces work. The two criteria are calculated for a displacement of one meter. During the optimization process, the constraints on the ground reactions, on the validity of impact, on the torques, on the joints velocities and on the motion velocity of the robot prototype are taken into account. Simulation results are presented for the three gaits. All motions are realistic. *Curvet* is the less efficient gait with respect to the criteria studied. For slow motion, *trot* is the more efficient gait. But *amble* permits the fastest motion with the same actuators.

Keywords: walking robot - dynamically stable gaits - optimal trajectories – algebraic optimization – kinetic momentum – cyclic motion

1. Introduction

This paper takes interest in optimal reference trajectories for a quadruped robot without feet, using dynamically stable gaits.

Generally, a robot must walk along a path with a given average velocity, but the motion of its joints is not defined by this task. Thus, in order to define them, several techniques exist. Some of these are: (i) Use of arbitrary functions as Van der Pol's oscillators [1], cycloid functions [2], polynomial functions *etc* (ii) choice of motion deduced from the motion of human or animals [3] (iii) optimal motions. We choose the last strategy because the characteristics of the prototype and its actuators limits can be taken into account.

There are different optimization techniques. Pontryagin principle can be used [4]. The problem can be transformed into an algebraic optimization problem imposing that some variables be described by a parameterized function. Only sub optimal solutions can be defined but the problem is drastically simplified.

The choice of optimization variables is not unique. The torques, the Cartesian coordinates or joint coordinates can be used. In [5], the optimization parameters are piecewise constant torques. Different authors [6-7] have imposed polynomial approximations for the Cartesian coordinates of the hip, the swing foot and the platform angle. To avoid the use of inverse dynamic model and inverse geometric model, we prefer to use the joint coordinates as optimized variables. In [8], it is also mentioned that the joint coordinates have to be chosen because their dynamics are relatively slow with respect to the actuator dynamics.

The joint variables can be described using Fourier series decomposition [8], polynomial functions [6-9] *etc*. We use coefficients of polynomial functions (or more exactly some configurations and velocities which define these coefficients) as optimization variables.

Our goal is to generate optimal cyclic reference trajectories for a quadruped robot employing three kinds of walking gaits: *curvet* – *amble* – *trot*. The studied quadruped has non-actuated joint; a passive ball joint models the punctual contact between the leg tip and the ground.

The second section introduces the representation of the walking robot and the studied gaits. Section 3 is devoted to the mathematical modeling. Section 4 presents the optimization problem. The optimization method is developed in section 5. Section 6 shows some simulation results. At least, in section 7, main conclusions are drawn from this study.

2. Representation of the quadruped

Our walking robot is composed of a platform and four legs. Each leg is made up of a thigh and a shin. Each thigh is connected to the platform by a one-degree-of-freedom (d.o.f) rotating haunch joint and, to the shin by a one-degree-of-freedom knee joint. The lengths of the thighs and the shins are *15* cm. However their masses are different: *1.6* Kg, for the thigh and *0.2* Kg for the shin. The length and the width of the platform are *37.4* cm and *30.0* cm respectively. The quadruped mass is *13.85* Kg. The axes of all joints are parallel to the transverse axis of the quadruped platform. The platform geometric center coincides with the platform mass center and is located in the plane of the haunch joints (see Figure 1a). Each link is assumed massive and rigid.

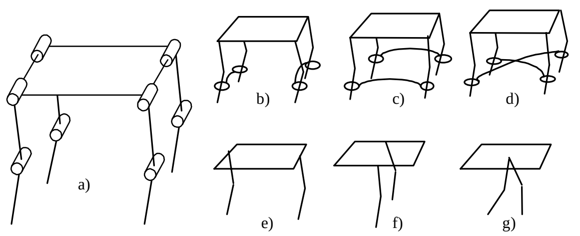


Figure 1: Description of the robot and studied gaits.

2.1. The studied gaits

The motions studied involve simultaneous motion of two legs. So, we have three types of gait: *curvet* (front and rear legs move together - Figure 1b), *amble* (legs on the same side move together - Figure 1c) and *trot* (diagonal legs move together - Figure 1d).

For all gaits, no flight is allowed. A passive ball joint models the contact between one leg tip and the ground, which means that there are not feet.

In order to simplify the dynamic model we use the virtual legs introduced by Raibert [11]: one virtual leg symbolizes two legs with simultaneous motion. The use of virtual legs implies that the behavior of the two real legs in simultaneous motion is identical. The dimension of each virtual leg is the same as those of the real legs. But the virtual leg is twice heavy. The ground reaction applied to a virtual leg tip is the sum of forces applied in the two real legs. The same rule exists for the joint torques.

2.2. Curvet

In the locomotion called *curvet* here, both fore legs move identically with respect to the platform, and both hind legs move identically as well. The fore legs (respectively the hind legs) of our quadruped are coupled (Figure 1b). At least two legs (fore or hind) are always on the support, because our *curvet* does not contain any flight phase. Therefore, the transverse axis of the platform is always horizontal, and the platform rotates around this axis only.

For this gait, one virtual leg is located in the middle of the two fore legs and the second one is located in the middle of the hind legs (Figure 1e).

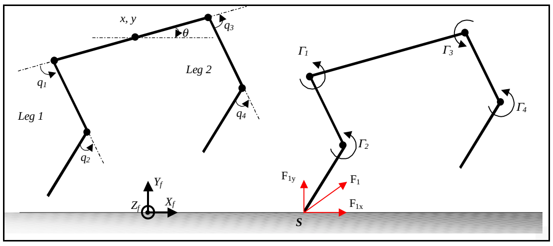


Figure 2: Generalized coordinates and forces of the quadruped for *curvet.*

The five-link model of a quadruped for *curvet* gait is planar (in sagittal plane). It contains the platform and the two virtual identical legs (Figure 2). The global situation of the platform is described by θ, the absolute orientation angle of the platform with respect to the horizontal and by x and y, the coordinates of the platform mass center. The configuration of the legs is defined by [q_1, q_2, q_3, q_4] (Figure 2). These configuration variables of the quadruped can be grouped into $n_q=5$ joint and orientation variables $q=[q_1, q_2, q_3, q_4, \theta]$ and $n_p=2$ position variables $p=[x, y]$. The seven generalized coordinates of the vector X, $X=[p, q]$ describe the configuration of this mechanism in the vertical plane X_fY_f. Four torques act on the leg joints $\Gamma=[\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4]$, $n_t=4$. Let F_i be the external forces applied to the leg tips i. When the leg i is a swing leg, the force F_i is zero. The dimension of F_i is $n_c= n_p=2$, $F_i = (F_{ix}, F_{iy})$ because the model is planar.

It can be proved using symmetry consideration and the global equilibrium equations that a solution for the model with four real leg can be deduced from the solution obtained for the model with two virtual legs: the torques in the joint of each real leg are half of the torques in the joint of the corresponding virtual leg and the reaction force in the real leg tip is half of the reaction force in the virtual leg tip.

2.3. *Amble*

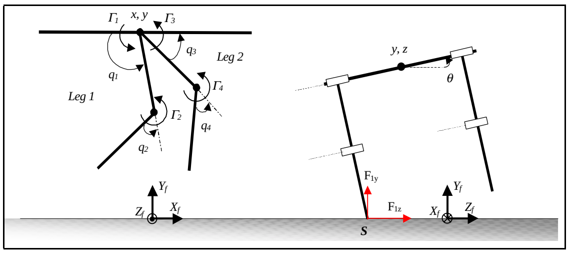


Figure 3: Generalized coordinates and forces of the quadruped for *amble.*

For the *amble* gait, both right legs (resp. both left legs) move always identically with respect to the platform. This means that right legs (left legs) are coupled (Figure 1c). At least two legs (left or right) are always on the support because the gait does not include flight phase. Therefore, the longitudinal axis of the platform is always horizontal and the platform rotates around this axis only.

A simple model with two virtual legs can be considered for the *amble* gait. One of the virtual legs is in the middle of the two left legs and represents them. The second one is in the middle of the two right legs (Figure 1c, 1f).

The side view (with horizontal platform) and the front view (with straight and parallel legs) of the robot are drawn in Figure 3. The Cartesian coordinates of the platform mass center are x, y, z. The angle between the platform and the horizontal plane is θ. The configuration variables are $n_q=5$ joint and orientation variables $q=[q_1, q_2, q_3, q_4, \theta]$ and $n_p=3$ position variables $p=[x, y, z]$. The eight generalized coordinates $X=[p, q]$ describe the configuration of this mechanism in the Cartesian space $X_fY_fZ_f$. $\Gamma=[\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4]$ denotes the torques acting on the same joints as in the *curvet*, $n_t=4$. But unlike the *curvet*, the external forces applied to the leg tips for the *amble* gait have $n_c=n_p=3$ components $F_i = (F_{ix}, F_{iy}, F_{iz})$.

From the solution obtained for the model with virtual legs, it can be shown that a solution for the model with four real leg is such that: the torques in the joints of each real leg are half of the torques in the joints of the corresponding virtual leg and the sum of the reaction forces in the real leg tips is the reaction force of each virtual leg. But the distribution of the reaction forces between the two real supporting legs cannot be deduced using the virtual model.

2.4. *Trot*

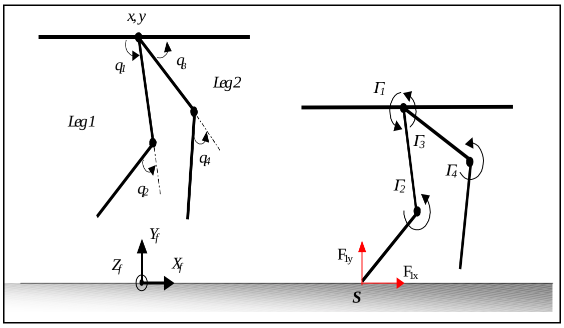


Figure 4: Generalized coordinates and forces of the quadruped for *trot.*

For the *trot* gait, the diagonal legs move identically with respect to the platform, as it is shown in Figure 1d. Moreover, we suppose that always two or four legs are on the support.

A simple model with two virtual legs, connected to the platform at its mass center (Figure 1g) can be introduced to describe the *trot* gait. The coupled diagonal stance legs can be represented by one virtual leg (Figure 1d), because the absolute velocities of their links are always identical. But the coupled diagonal swing legs can be represented by one virtual leg only when the platform orientation is constant. If the platform orientation is not constant, the absolute velocities of their links are not identical, and consequently the motion equations differ for the models with four real legs and with two virtual legs respectively. When both diagonal coupled swing legs touch the support simultaneously, the platform must be horizontal. Therefore, we impose that the platform is always horizontal during the trotting (Figure 1g) in the model with two virtual legs. So, we can speak of equivalence between the models with four legs and two virtual legs only for trotting with horizontal platform [13]. Note that for a horse, its platform is always horizontal during the *trot* [12].

If the platform of the model with virtual legs is initially horizontal and if both initial angular velocity and the sum of the torques around the platform mass center are zero, then the platform will be horizontal all the time. The last condition implies that:

$$\Gamma_1 = -\Gamma_3 \qquad (1)$$

This previous equality is a necessary condition for a horizontal motion of the platform.

The motion of the described model with two virtual legs and horizontal platform is planar. The scheme of the robot in the *trot* gait is in Figure 4. The configuration variables are $n_q=4$ joint and orientation variables only $q=[q_1, q_2, q_3, q_4]$ because the platform is horizontal and $n_p=2$ position variables $p=[x, y]$. So, a vector of six generalized coordinates $X=[p, q]$ describes the configuration of our quadruped in the vertical X_fY_f plane. The torque must satisfy the relation (1), so only three torques are independent and the vector Γ is defined by $\Gamma=[\Gamma_2, \Gamma_3, \Gamma_4]$, $n_t=3$. The external forces applied to the leg tips have $n_c= n_p =2$ components as for the *curvet* gait: $F_i = (F_{ix}, F_{iy})$.

It can be proved using symmetry consideration and the global equilibrium equations that a solution for the model with four real legs can be deduced from the solution obtained for the model with two virtual legs: the torques in the joint of each real leg are half of the torques in the joint of the corresponding virtual leg and the reaction force in the real leg tip is half of the reaction force in the virtual leg tip.

2.5. Decomposition of a step

For all studied gaits, the motion of the quadruped, is defined by the following cycle: an instantaneous double support phase (DS), a single support phase on leg 1 (SS_1), an instantaneous double support phase (DS), a single support phase on leg 2 (SS_2).

In the *curvet* gait, the fore legs are always in front of the hind legs and each half step must be calculated. The *curvet* gait is decomposed into four successive phases (Figure 5):

- a single support phase (SS_1) on the rear leg (leg 1) (Figure 5-a),
- an instantaneous double support 1 (Figure 5-b),
- a single support (SS_2) on the front leg (leg 2) (Figure 5-c),
- an instantaneous double support 2 (Figure 5-d), etc…

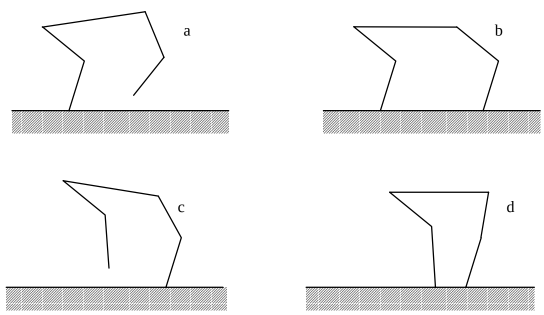


Figure 5: Description of the *curvet* gait.

In the *amble* and *trot* gaits, at the beginning of SS the transferred leg is behind the supporting leg. At the end of this phase the transferred leg is in front of the supporting one. The trajectories are defined such that the legs swap their role from one half step to the following one. Thus the behavior on the second step can be deduced from the behavior on the first step without calculation.

3. Dynamic model

3.1. Contact equation

The robot is modeled by rigid links interacting with rigid ground. Its configuration is described by the n_p+n_q variables X. A leg remains on the support if the position of the leg tip is constant (no take-off, no sliding). These relations can be written as:

$$d_i(X) = constant \tag{2}$$

$i=1, 2$ depending on the number of the leg in support. d_i is a n_c vector function with $n_c=2$ for planar motion involved in *curvet* or *trot* and $n_c=3$ for *amble*.

The linear velocity and acceleration of the leg tip are zero:

$$D_i(q)^T \dot{X} = 0 \tag{3}$$

$$D_i(q)^T \ddot{X} + H_i(q,\dot{q}) = 0 \tag{4}$$

where D_i is the (n_c, n_p+n_q) Jacobian matrix of $d_i(X)$ and $H_i(q,\dot{q})$ contains the nonlinear terms obtained during the derivation.

3.2. Dynamic model

The general forces acting on the quadruped are the independent torques Γ and the reaction forces F_i (on leg i, $i=1, 2$).

The dynamic model is written using the Lagrange formalism. For Single Support on leg i (SS_i), the ground exerts the force F_i. Thus the dynamic model of the quadruped have the following form:

$$A(q)\ddot{X} + H(q,\dot{q}) = D_G \Gamma + D_i(q) F_i \tag{5}$$

where A is the (n_p+n_q, n_p+n_q) inertia matrix, H is the (n_p+n_q, 1) vector of Coriolis, centrifugal and gravity effects. D_i (n_p+n_q, n_c) and D_G (n_p+n_q, n_t) allow to take into account the effect of the external force and torques.

3.3. Free evolution of the robot

Taking into account constraint equation (4), there are $n_p + n_q - n_c = n_q$ (because $n_p=n_c$) d.o.f. in SS phase. A dynamic model expressed only as function of vector q and its derivative can be written:

$$A_q(q)\ddot{q} + H_q(q,\dot{q}) = D_q\Gamma \qquad (6)$$

where A_q is the (n_q, n_q) inertia matrix, H_q is the (n_q, 1) vector of Coriolis, centrifugal and gravity effects, D_q (n_q, n_t) allow to take into account the torques.

There are only n_t independent actuators but n_q independent configuration variables.

For the three gaits studied:

$$n_q\text{-}n_t=1 \qquad (7)$$

So the degree of under-actuation of the system in SS is 1. Only the evolution of n_t variables can be chosen, the evolution of the other orientation variable can be deduced from this choice as it will be shown.

A new notation is introduced; the vector q is decomposed into a vector of n_t components denoted Q and a scalar denoted β. In the *curvet* and *amble* gaits, $Q = [q_1, q_2, q_3, q_4]$ and $\beta = \theta$. In the *trot* gait, $Q = [q_2, q_3, q_4]$ and $\beta = q_1$.

The projection, along the direction Z_f for *curvet* and *trot* and along the direction X_f for *amble*, of the kinetic momentum around the supporting leg tip S depends only on the gravity effects. This projected kinetic momentum around S of the quadruped is linear with respect to $\dot{\beta}$:

$$\sigma = f_1(q)\dot{\beta} + f_2(q,\dot{Q}) \qquad (8)$$

The external forces are the gravity effects and the ground reaction force applied on S. Thus the theorem on the total angular momentum changing around the point S can be written:

$$\dot{\sigma} = -Mg\{c(q)\} \qquad (9)$$

where M is the mass of the robot, g is the gravity acceleration, $c(q)$ is the distance between the quadruped mass center and point S along the axis X_f for the *curvet* or *trot* gaits and along axis Z_f for the *amble* gait.

The time derivative of expression (8) and the equation (9) permit to define the acceleration of β as function of the acceleration of Q and of the position and velocity of the robot:

$$\ddot{\beta} = F(q,\dot{q},\ddot{Q}) \qquad (10)$$

When: $f_1(q) \neq 0$

3.4. Impact with the ground

When the leg j touches the ground at the end of SS_i, an impact exists. This impact is assumed instantaneous and inelastic. This means that the velocity of the foot j touching the ground is zero after the impact.

The velocities just before and just after impact are noted $\dot{X}^-$ and $\dot{X}^+$ respectively; I_{F_i} and I_{F_j} denote the impulsive forces on the foot i and j respectively. The passive impact equations are [10, 14]:

$$\begin{cases} A(q)(\dot{X}^+ - \dot{X}^-) = D_j(q) I_{F_j} + D_i(q) I_{F_i} \\ D_j(q)\dot{X}^+ = 0 \\ D_i(q)\dot{X}^+ = 0 \end{cases} \tag{11}$$

Or $$\begin{cases} A(q)(\dot{X}^+ - \dot{X}^-) = D_j(q) I_{F_j} \\ D_j(q)\dot{X}^+ = 0. \end{cases} \tag{12}$$

The use of (11) or (12) depends on whether leg i remains on the ground or not. Since the impulsive forces must be directed upward, these systems admit generally one solution only. It can be shown that if the case of sliding is also introduced, only one solution exists [15]. With these equations, the velocity after the impact can be calculated when the velocity before the impact is known.

4. Definition of the problem

4.1. Algebraic optimization

Generally, the robot must walk along a path with a given average velocity but the motion of its joints is not defined by this task. Reference trajectories minimizing some criteria for some given velocities are defined. These trajectories can be used to test the design of a robot and its actuators [16] or in the control law. The optimization problem is solved in this paper in an algebraic way. The evolution of the variable Q is defined as a time polynomial function. The optimized parameters are the coefficients of these polynomials. Constraints equations are added to impose convenient cyclic evolution of the robot.

4.2. Constraints

The context of this study is to define an optimal trajectory for a given quadruped with given actuators. In particular the considered constraints on the actuators are the maximum torques (Γ_{max}) and maximum velocities (V_{max}).

Some other constraints must be checked to insure that the optimal trajectory is possible:

- the reaction force must be directed upward,
- the ratio between the horizontal and vertical force must be less than the friction coefficient,
- the swing leg tip must not touch the ground before the prescribed time,
- some constraints corresponding to the validity of impact are also taken into account,

All these constraints can be easily written as inequality conditions. The average motion velocity of the quadruped is given and is considered as an equality constraint.

4.3. Criteria

In the optimization process we consider two criteria.

The *torque cost* is defined as the integral of the norm of the torque [5] for a displacement of one meter:

$$C_1 = \frac{1}{x(T) - x(0)}\left(\int_0^T \Gamma^* \Gamma \, dt\right) \tag{13}$$

The *energy cost* is defined as the integral of the absolute value of the work of external forces [17] for a displacement of one meter:

$$C_2 = \frac{1}{x(T) - x(0)}\int_0^T |\Gamma|^* \, |\dot{q}| \, dt \tag{14}$$

In electrical motor, neglecting the friction, and for a cycle of walk, most part of the energy consumption is due to the loss by Joule effect. Criterion C_1 is proportional to this loss of energy. It characterises the energy that must produce the battery during the motion. The criterion C_2 is less dependent on the driving actuator. It characterises the variation of mechanical energy of the system. It is defined by assuming that the negative work produced by the actuator to slow down cannot be used by another one or during the acceleration phase. No brake and no spring are used so the negative work must be produced by the actuators. Therefore the absolute value of the work is considered. The units of C_1 and C_2 are respectively N^2ms and N.

4.4. Local optimization

In order to solve a nonlinear constrained problem, we choose to use a classical local optimization method SQP (Sequential Quadratic Programming). In this method, a Quadratic Programming (QP) algorithm is solved at each iteration. An estimate of the Hessian of the Lagrangian is updated at each iteration using the BFGS formula [18]. The algorithm *constr* from the package *Matlab* is used.

5. The gait optimization method

The gait studied is composed of stance phases. A passive impact phase separates the stance phases. Two following half steps must be considered for the *curvet* gait.

5.1. The optimized variables

During the SS_i phase, we choose to define the evolution of variables Q, as a four-order polynomial function of time,

$$Q(t) = a_0^i + a_1^i t + a_2^i t^2 + a_3^i t^3 + a_4^i t^4 \tag{15}$$

These polynomials connect specified initial and final configurations and velocities and an intermediate configuration ($Q_i^i, \dot{Q}_i^i, Q_f^i, \dot{Q}_f^i$ and Q_{int}^i). The indices *i, f* and *int* correspond respectively to initial (at $t = 0$), final (at $t = T$) and intermediate (at $t = T/2$) states. The exponent i will take the value of the number of the legs in support. The polynomial functions *Q(t)* and its derivatives are uniquely defined, during $SS_{i,}$ using $Q_i^i, \dot{Q}_i^i, Q_f^i, \dot{Q}_f^i, Q_{int}^i$ and T^i .

Four order polynomials are used because a smaller order does not permit in some case to obtain a feasible solution. A higher order gives a larger number of optimization variables, the local optimization is not facilitated, and the set of optima is larger.

When $Q(t)$ is defined, the theorem of total kinetic momentum is used to define the evolution of β with Eq.(10). The orientation and angular velocity of β can be deduced, if they are known at a given time because equation (10) defines the acceleration of β. We choose to define them at the end of the single support phase: β_f^i and $\dot{\beta}_f^i$. This point will be detailed in section 5.2.2.

In summary, if the motions of the legs are defined and if $\beta_f, \dot{\beta}_f$ are known, then the evolution of vector q and its derivatives can be deduced. The evolution of X and its derivatives can be calculated with the contact equations (2), (3), (4). Then the torque and reaction forces corresponding to the motion are defined by the dynamic model (5). In the three gaits studied, this model is over-determined. But this system is compatible and admits a unique solution because the evolution of β satisfies Eq.(10). Then the criterion and constraints can be evaluated.

5.2. Definition of cyclic motion

In fact, the desired trajectory has the particularity to be cyclic: two following steps must be identical. The conditions of periodicity are easily satisfied for *Q*. Some explicit conditions of periodicity will be written; the number of optimization parameters will be reduced. For the evaluation of β, the condition of cyclicity can not be solved explicitly, this condition will be written as an equally constraint.

5.2.1. Continuity for successive single support phases

Since the position of the robot is constant during the instantaneous passive impact (touch down configuration) the following condition can be written:

$$Q_f^i = Q_i^j \quad \beta_f^i = \beta_i^j \quad i = 1,2;\ j=1,2;\ i \neq j \tag{16}$$

The initial and final configurations are double support configurations. Indeed, the two leg tips are on the ground. We impose without loss of generality that the leg tip 1 for the initial DS is located at the origin of an absolute frame.

$$d_1(X) = 0 \tag{17}$$

One equation can be added because the vertical coordinate along Y_f of the leg tip 2 must be zero (for a motion on a horizontal surface). So the number of independent variables to describe the double support configurations is only $n_p + n_q - n_c - 1 = n_t$ variables. Thus if Q_f^1 and Q_f^2 are defined β_f^1 and β_f^2 can be deduced.

The velocity just after the impact is calculated using equations (11) or (12) and the knowledge of the velocity of the robot before the impact. Thus the velocity of the robot after the impact can be defined as:

$$\dot{Q}_i^i = g_Q(Q_f^j, \dot{Q}_f^j, \beta_f^j, \dot{\beta}_f^j)\ ,\ i = 1,2;\ j=1,2;\ i \neq j \tag{18}$$

$$\dot{\beta}_i^i = g_Q(Q_f^j, \dot{Q}_f^j, \beta_f^j, \dot{\beta}_f^j)\ ,\ i = 1,2;\ j=1,2;\ i \neq j \tag{19}$$

Using all these continuity equations, the optimization variables can be reduced to:

$Q_f^1, \dot{Q}_f^1, Q_{int}^1, T^1, \dot{\beta}_f^1, Q_f^2, \dot{Q}_f^2, Q_{int}^2, T^2, \dot{\beta}_f^2$.

For the *curvet*, twenty-eight optimization variables define an optimal motion because *Q* is a vector of $n_t=4$ variables.

For the *amble* and *trot*, we impose a symmetric role of the two legs between two following half steps. The evolution on the second half step can be deduced from the evolution on the first half step. The optimization variables are reduced to: $Q_f^1, \dot{Q}_f^1, Q_{int}^1, T^1, \dot{\beta}_f^1$.

For the *amble*, fourteen optimization variables define an optimal motion (Q is a vector of $n_t=4$ variables).

For the *trot*, eleven optimization variables define an optimal motion (Q is a vector of $n_t=3$ variables).

The set of optimization variables is denoted Θ. It defines completely $Q(t)$ and includes $\dot{\beta}_f^1, \dot{\beta}_f^2$.

5.2.2. Evolution of β

For a given value of Θ, the behavior of β can be defined by the dynamic equation (10). So we can write for each SS_i:

$$\ddot{\beta}^i(t) = F_1^i(\beta(t), \dot{\beta}(t), \Theta, t) \tag{20}$$

This equation can be integrated numerically because the initial position and velocity of β are known using (16) and (19). The position and velocity of β can be calculated as a function of time t for each SS_i:

$$\dot{\beta}^i(t) = F_2^i(\Theta, t) \tag{21}$$

$$\beta^i(t) = F_3^i(\Theta, t) \tag{22}$$

The defined trajectory is cyclic if and only if these two equations evaluated at $t = T^i$ give the expected orientation and angular velocity of β for each SS_i:

$$\dot{\beta}_f^i = F_2^i(\Theta, T^i) \tag{23}$$

$$\beta_f^i = F_3^i(\Theta, T^i) \tag{24}$$

To conclude, the evolution of the robot is completely defined by the values of:

$$\Theta = Q_f^1, \dot{Q}_f^1, Q_{int}^1, T^1, \dot{\beta}_f^1, Q_f^2, \dot{Q}_f^2, Q_{int}^2, T^2, \dot{\beta}_f^2$$

but these parameters must be chosen in order to satisfy (23) and (24). This problem is solved numerically.

6. Results analysis

In this section two types of results are presented: (i) typical optimal motion for the three gaits and with the minimization of both criteria, (ii) evolution of the cost functions versus the motion velocity of the quadruped.

In the first part of the analysis (sections 6.2 and 6.3), optimal motion are presented for a motion velocity equal to *0.6* m/s. (*2.16* km/h). For all the numerical tests each figure includes the stick-diagram (i) of the step (the unit is meter), (ii) the torque versus velocity for the knee and haunch actuators, the ground reactions and torques in function of the time during the step. The torque versus velocity plots highlight the respect of the actuators constraints, the margin available to react to disturbance, as well as the maximal torque used for the motion. For the *curvet* two half steps are presented. For the *amble* and *trot*, only one half step is shown because the second one can be easily deduced. Some comments on the effects of the choice of the criterion are given in section 6.4.

6.1. The constraints

For this study, the physical parameters of a prototype with two virtual legs under construction are used. The following constraints are taken into account.

The actuators are able to produce *0.8* N.m as maximal torque. The maximal velocity of the knee actuators must be the follow domain: *6600* tr/mn for a null torque and 4200 tr/mn for a torque equal to 0.8 N.m . The limits on the haunch actuators are *5000* tr/mn for a null torque 0.0 N.m and *3300* tr/mn for a torque equal to 0.8 N.m . These constraints are shown in Figure 6. The gear ratio for the actuators is *50*. The minimal vertical reaction force is *10* N. The chosen friction coefficient is *1*. The evolution of the free leg tip must be higher than a parabolic function with a maximum of 5.10^{-3} m.

We impose that the time of both single support phases is included between:

$0.05\ \text{s} < T_{ss_i} < 0.5\ \text{s}$, for the *curvet* $\quad (i = 1, 2)$

$0.05\ \text{s} < T_{ss} < 2\ \text{s}$, for the *trot* and the *amble*

6.2. Some optimal motions with respect to the energy criterion

6.2.1. A curvet gait

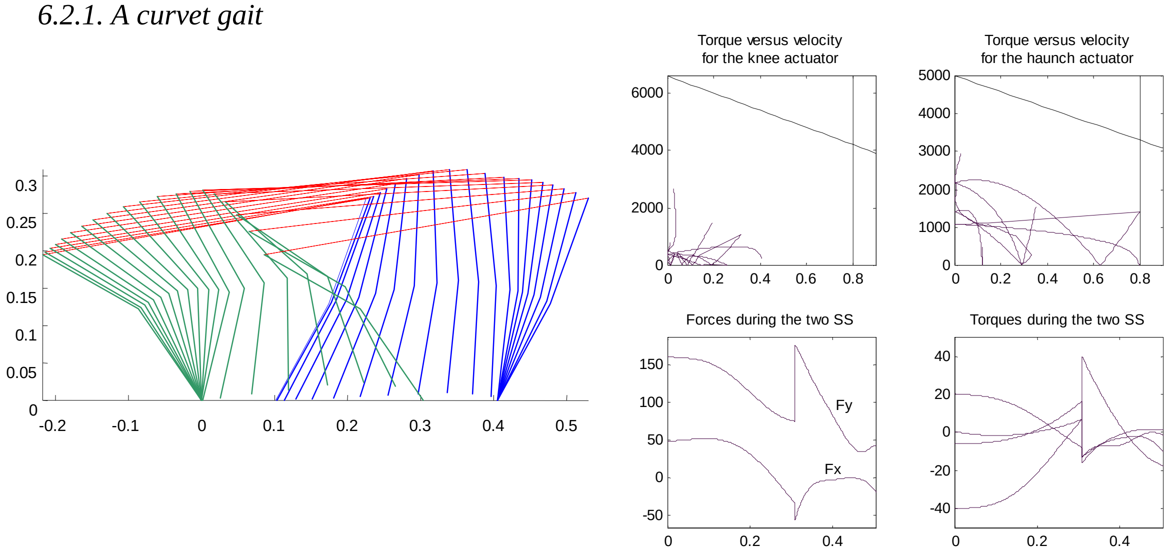
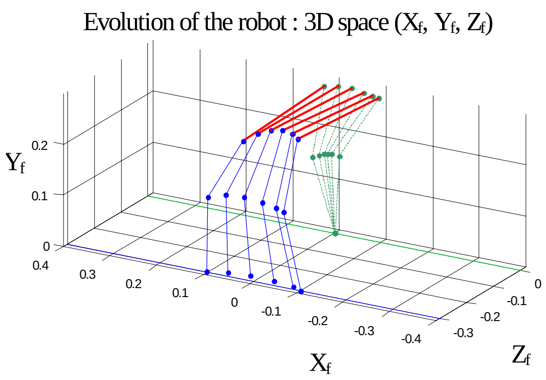
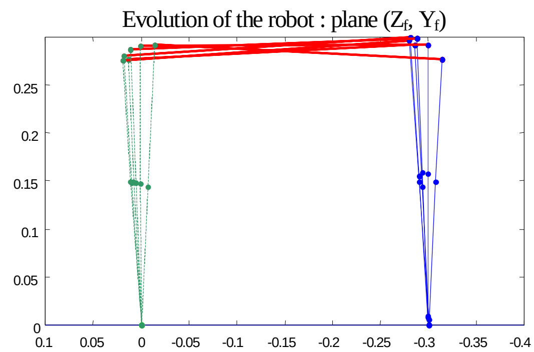


Figure 6: *Curvet* simulation results with minimization of the energy cost.

The results are presented in Figure 6. The value of the energy criterion is *78.87* N. The durations of the first and second half steps are *0.31* s and *0.196* s respectively. The minimal vertical reaction force is *30* N. For this walk with minimization of energy, the actuator torques are relatively low for the knee. The constraint on torque is active only for the haunch actuator of the stance leg for both half steps.

6.2.2. An amble gait





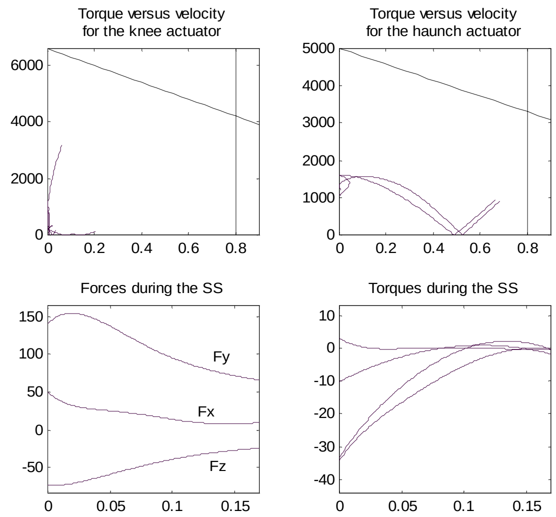


Figure 7: *Amble* simulation results with minimization of the energy cost.

The stick-diagram of the robot in the 3D space $X_fY_fZ_f$, is presented in Figure 7: the length of the platform is reduced to zero in order to facilitate the visualization of the robot motion. The value of the energy cost criterion is *55.13* N. The duration T of the step is *0.17* s. The minimal vertical reaction force is close to *70* N. The constraint on the torques are never active during the step. The potentialities of the actuators are under-used in the domain velocities/torques.

6.2.3. A trot gait

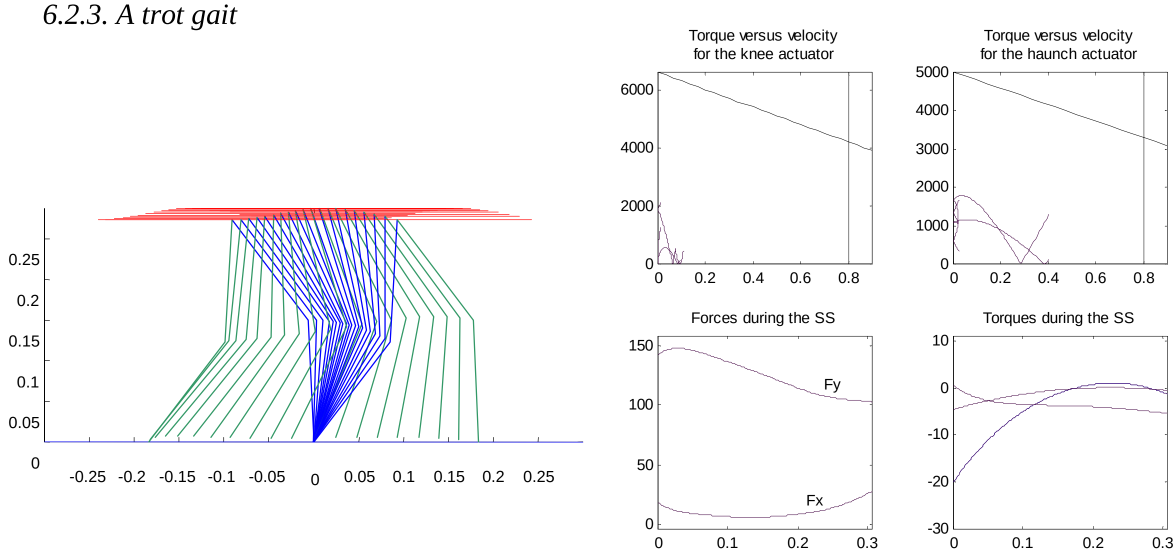


Figure 8: *Trot* simulation results with minimization of the energy cost.

The results are presented in Figure 8. The value of the energy cost is *29.98* N. The duration *T* of the half step is *0.31* s. The minimal vertical reaction force is close to *100* N at the end of the step. The actuators are under-used. The altitude of the swing leg tip is close to the value defined by the parabolic function constraint that sometimes becomes active.

6.3. Some optimal motion with respect to the torque criterion

6.3.1. A curvet gait

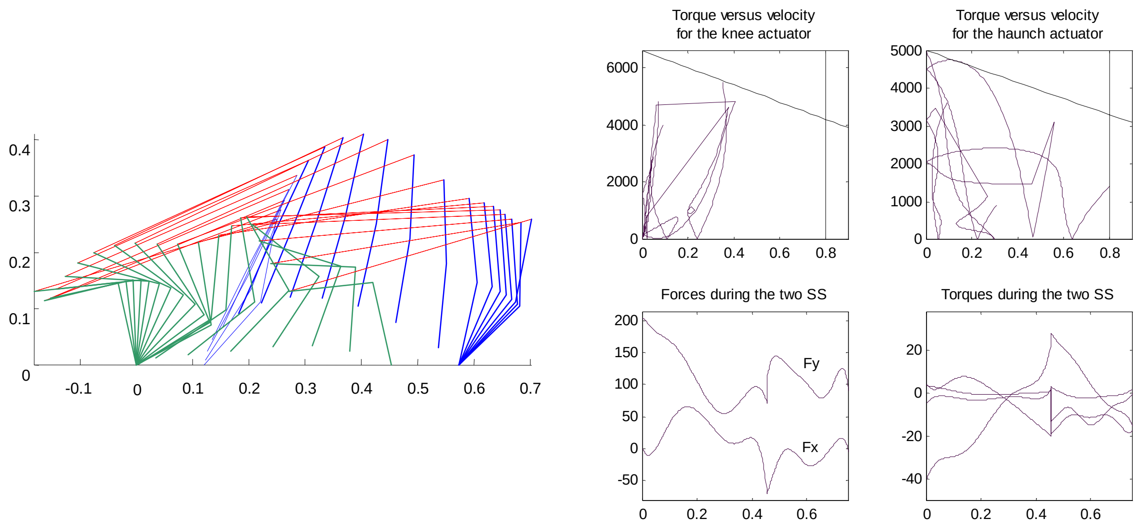
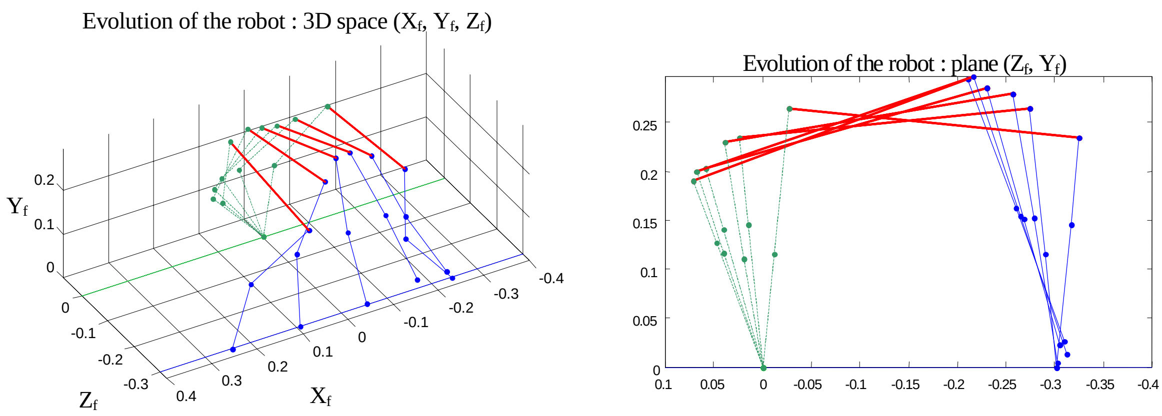


Figure 9: *Curvet* simulation results with minimization of the torque cost.

In the stick-diagram of Figure 9 the magnitude of the evolution of the platform orientation is more important than for the previous case. The value of the torque cost criterion is *802.26* N^2ms. The times for the first half step are *0.456* s, and *0.298* s for the second one. The minimal vertical reaction force is close to *50* N. For this walk, the joint velocities are higher than for the energy minimization. The active constraints are the hip joint velocities of the swing leg during the second half step and the torque of the haunch at the beginning of the first half step.

6.3.2. An amble gait



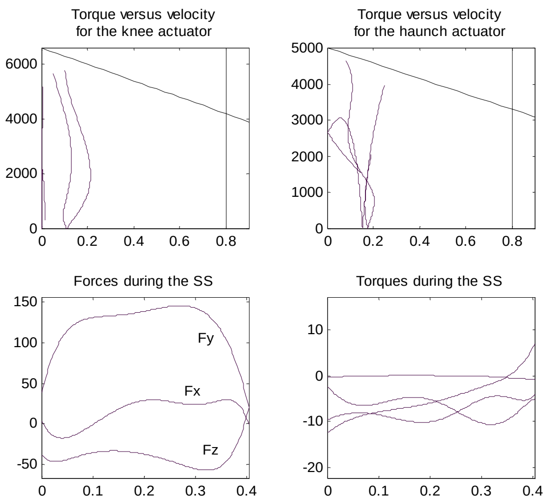


Figure 10: *Amble* simulation results with minimization of the torque cost.

The results are presented in Figure 10. The value of the torque cost criterion is *256,8* N^2ms. The duration T of the half step is *0.404* s. The minimal vertical reaction force is close to *25* N at the end of the step.

6.3.3. A trot gait

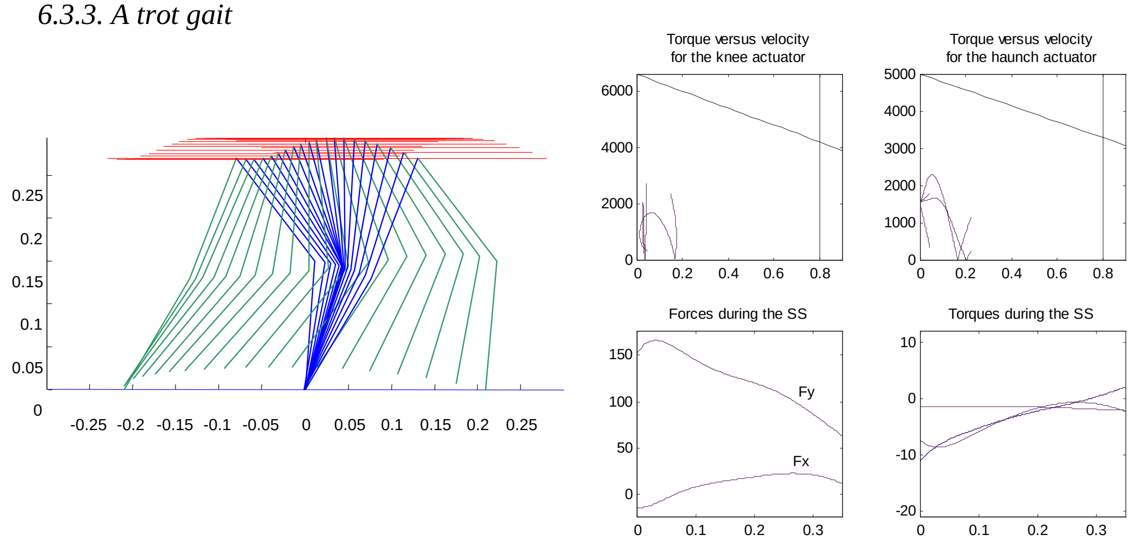


Figure 11: *Trot* simulation results with minimization of the torque cost.

The results are presented in Figure 11. The value of the torque cost criterion is *112.17* N^2ms. The duration T of the step is *0.35* s. The minimal vertical reaction force is close to *50* N at the end of the step. Here also the actuators are under-used.

6.4. Some comments on the optimal motion for a motion velocity equal to 0.6 m/s

With respect to both studied criteria the best gait is the *trot* and the worst is the *curvet*. This result was expected because a horse (our studied robot corresponds to a very simplified model of a horse), naturally uses a *trot* gait and never a *curvet* gait.

All the motions presented seem to be quite natural. The obtained motions are more realistic with energy optimization. These results are important because all the key configurations are defined by the optimization process but they are not imposed.

The choice of the criterion has influence on the optimal results. The characteristics of this influence are similar for all gaits and also for walking or running of a biped [9]. Their characteristics can be summarized as follows.

- The joint velocities are lower for the energy minimization than for the torque minimization,
- the joint torque are high at the beginning of the stance phase for energy minimization, but at the end of the stance phase, the joint torques are close to zero for most joints,
- as expected, the maximal torque is lower for the torque minimization than for the energy minimization.
- the amplitude of the change of platform orientation is larger with a torque minimization for *curvet* and *amble*, than for energy optimization.
- for energy optimization, the vertical position of the swing leg tip is very low and close to the constraint.

The optimization algorithms for the three gaits are programmed using Matlab. They converge to a local optimum. An efficient initialization for the optimization process is to choose the solution corresponding to the other criterion or for a velocity close to the tested velocity. The calculating time for optimization can vary between 5 hours in some cases of *trot* to 40 hours in some cases of c*urvet* (for a Pentium II 400Mhz PC*)*, mainly depending on the number of variables used in each gait but also on the criterion type and the number of actives constraints. This calculating time can be improved, by compiling part of the programs for example.

6.5. Evolution of the criterion cost versus motion velocity for the three gaits

For a set of velocities, the energy cost is compared for the three gaits, with both criteria cost in energy and torque (Figure 12 and Figure 13).

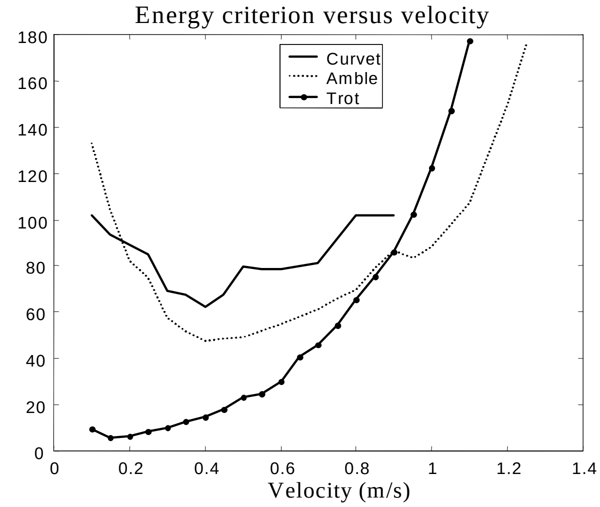


Figure 12: Energy cost criterion versus velocity for the three gaits.

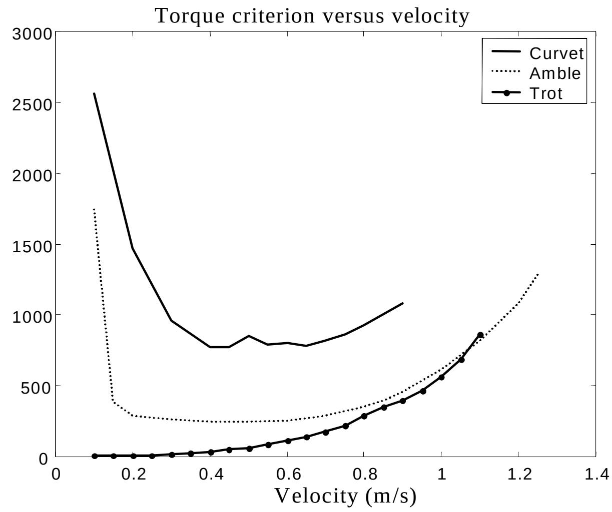


Figure 13: Torque cost criterion versus velocity for the three gaits.

The velocity limits for the three gaits are similar with both minimizations but differ for the same actuators depending on the gait used. For *curvet*, the maximal motion velocity is close to *0.9* m/s. For *trot*, the maximal motion velocity is close to *1.1* m/s and for *amble 1.3* m/s. The maximal velocities are reached with the *amble* gait.

Globally, the criteria cost value is greater for the *curvet* than for the other gait both for energy and torque minimization. This result is connected with the fact that no animal uses *curvet*.

For the low velocities, the *trot* is the best gait with respect to energy or torque criteria. The evolution of the criteria C_1 and C_2 for this gait increase regularly. It is not the case for the *curvet* and *amble* that are more "expensive" for very low velocities. With minimization of the torque cost, we see on Figure 13 that the joint torques are close to zero for the *trot* gait.

For large motion velocities of the quadruped, the *amble* becomes better. This modification appears after *0.9* m/s for minimization of energy cost and *1.1* m/s for minimization of the torque cost.

7. Conclusion

A methodology to design cyclic optimal trajectories for a complete step of walking was developed for a quadruped without actuated feet. Three kinds of gait were studied: *curvet*, *amble* and *trot.* In order to use classical optimization technique, a sequential quadratic programming, the optimal trajectory was defined by a set of parameters. The evolution of the actuated joints is designed as fourth order polynomial functions in time. The coefficients of the polynomials are the optimization variables. The number of the optimization variables is reduced by imposing a polynomial evolution, with the introduction of virtual leg, and by taking into account explicitly of the cyclicity condition. This choice allows using classical local optimization technique. But the gait studied involves free evolution of the robot with respect to the ground because only two points are in contact with the ground. The evolution of this non-actuated configuration variable is defined by integration of a part of the dynamic model. And a boundary problem is solved. The solving of the boundary process is included into the optimization process and this resolution needs many calculations. Thus, in spite of a well-posed problem, the calculations needed to solve the complete problem are important. Some inequality constraints were taken into account such as the limits on torque and actuated joint velocities, the condition of no sliding during motion and impact, some limits on the motion of the free leg. In the presented work, virtual legs are introduced to reduce the modeling complexity. For the *curvet* and *trot* gaits the optimal motion of the virtual robot can directly define the optimal motion for the real robot. But for the *amble* gait, as only the sum of the reaction forces is defined, the constraints on the reaction forces only act on the sum. Therefore it cannot be ensured that the constraints are achieved for each leg of the real quadruped. Thus the optimal result for the *amble* gait must be taken into account carefully.

Realistic optimal motions were obtained for the three gaits. It is an important result because the determination of the initial, intermediate and final configurations were not imposed but chosen by the optimization process. The *trot* gait is the best one for slow motions of the quadruped. The *amble* gait is efficient for faster motion. The *curvet* gait is the worst gait with respect to both studied criteria (torque or energy cost). This is related with the fact that this gait is not a natural gait for animals.

All the optimization tests quoted in this article are already carried out for the study of the walking biped robots [9] and the results show that the influences of the choice of the criterion are similar on quadruped and biped robots.

The gaits studied in this paper were composed of stance phases connected by instantaneous double support. The extension of the method to the case of walk with non-instantaneous double support and running will be done.

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